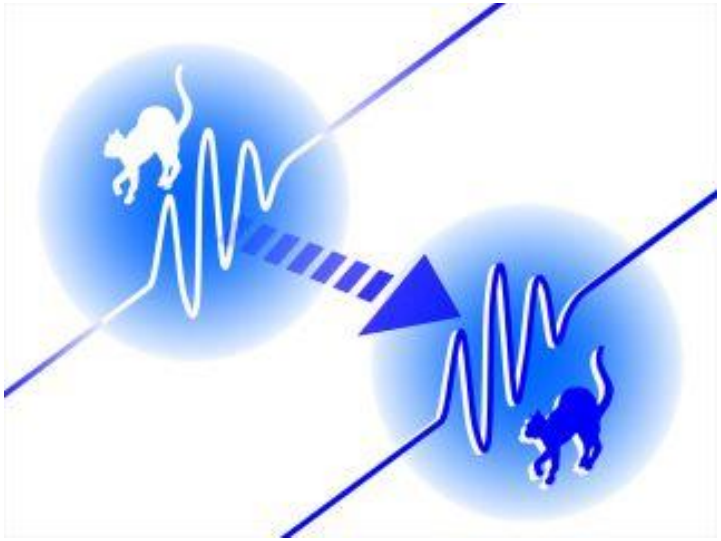
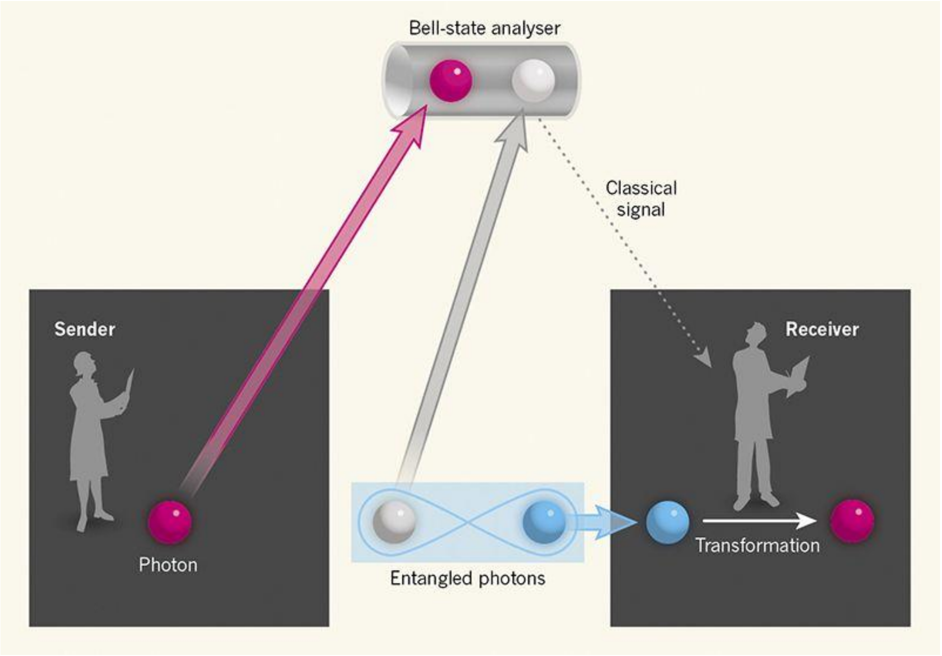
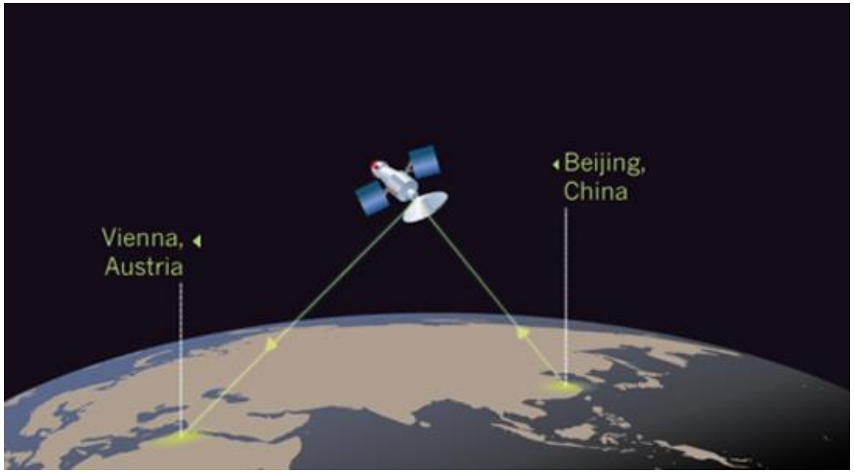
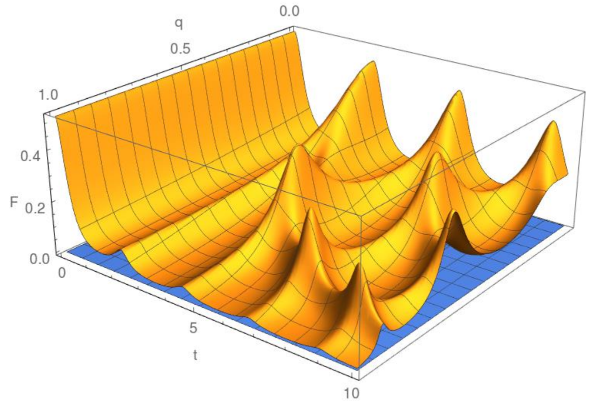
Calculation of Fidelity of Teleporation for two mode Gaussian states placed in a thermal bath









Outline

- Teleportation in art
- Quantum teleportation
- Recent experiments
- Discrete to continuous
- Our model
- Master equation
- System evolution
- Results
- Conclusions
- Thank you for your attention

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Aurelian Isar

Teleportation in art

Startgate

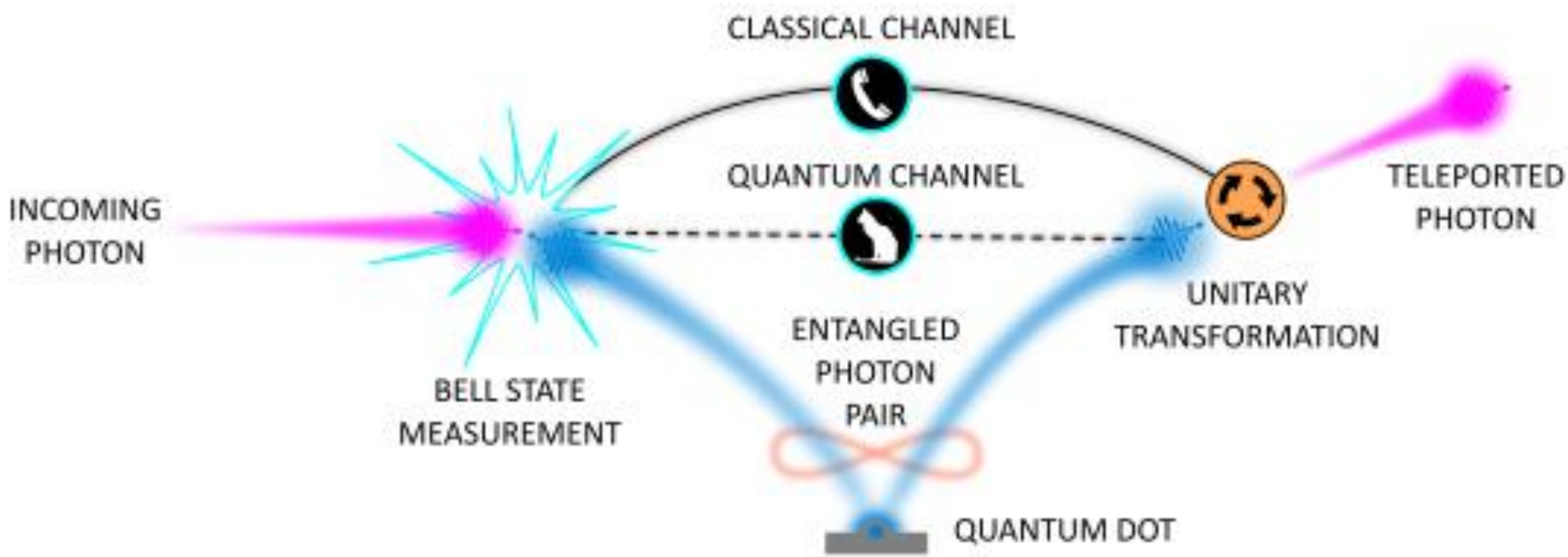
Startrack 1966

Warcraft

Harry Potter

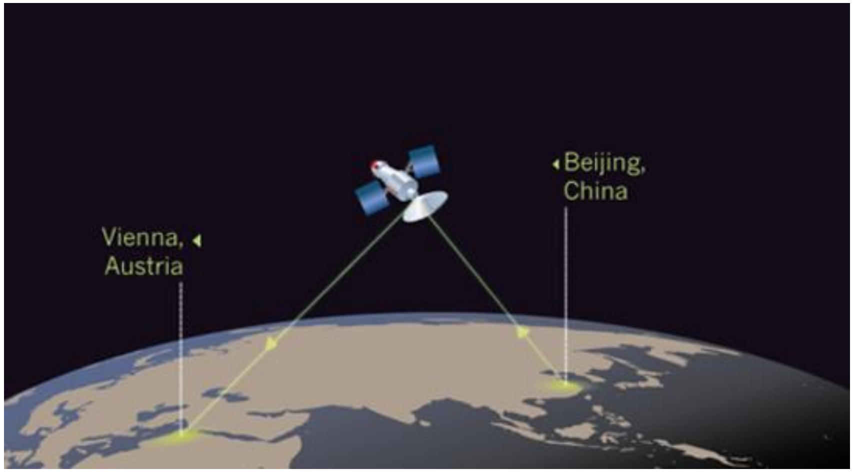
Startcraft

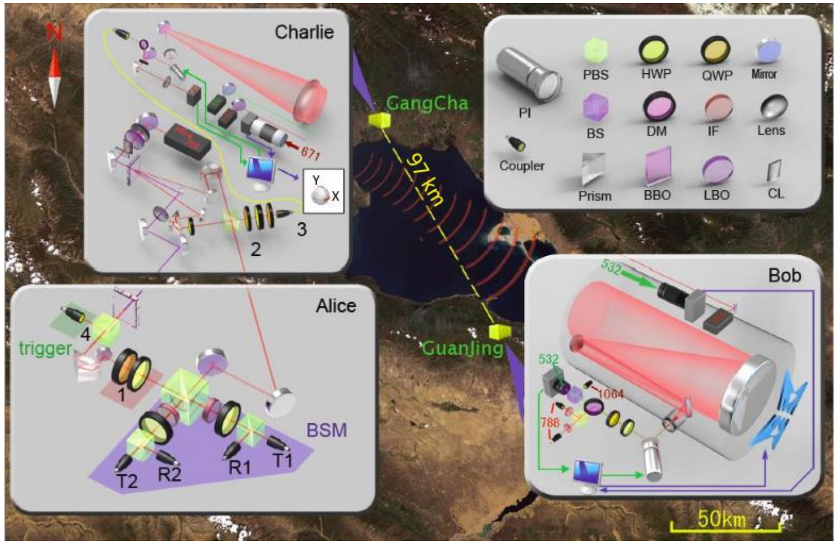
Quantum Teleportation

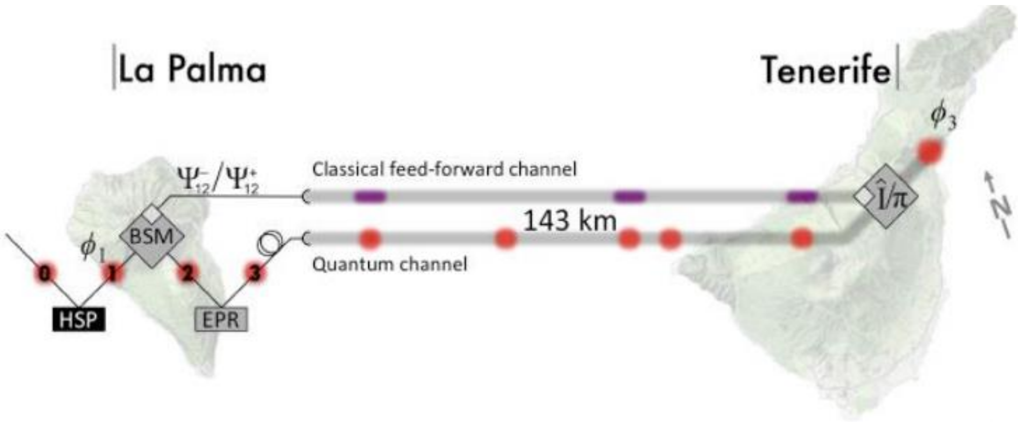


- Important technology for secure quantum communication and distributed quantum computing
- Transfers an unknown quantum state from A to B
- Requests a classical communication channel
- Requests an EPR pair

Recent experiments

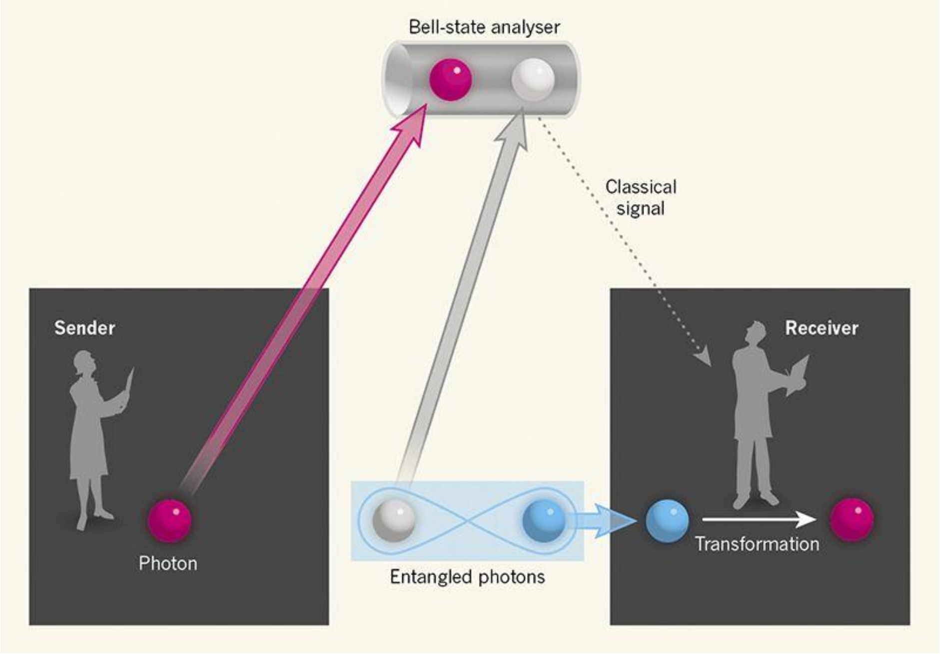


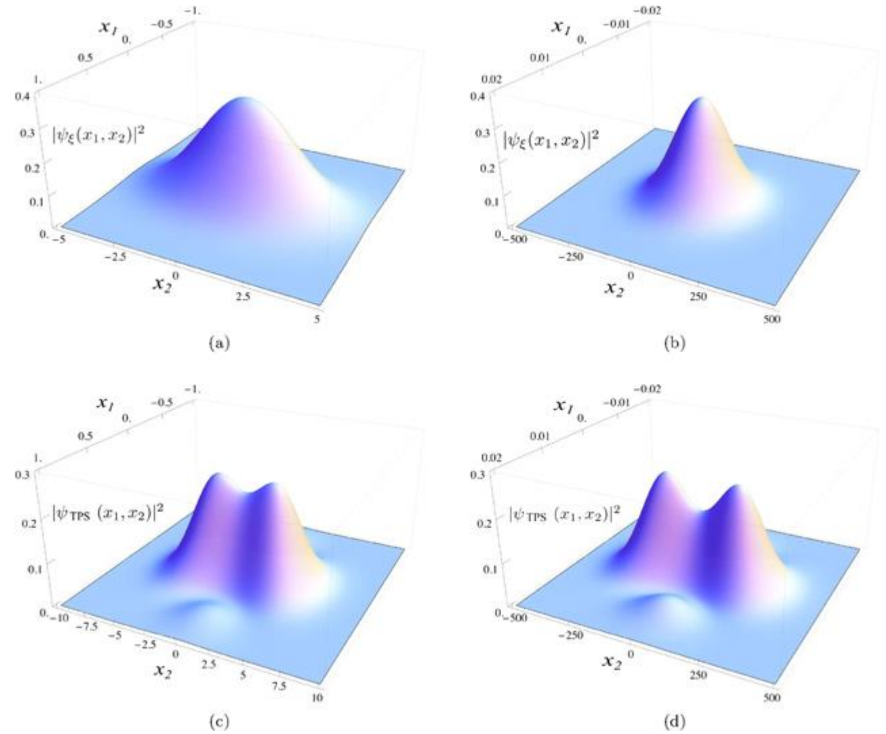




Ren, Ji-Gang, et al. "Ground-to-satellite quantum teleportation." Nature 549.7670 (2017): 70.

Discrete to Continuous





Seshadreesan, Kaushik P., Jonathan P. Dowling, and Girish S. Agarwal. "Non-Gaussian entangled states and quantum teleportation of Schrödinger-cat states." Physica Scripta 90.7 (2015): 074029.

Master Equation

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] + \frac{1}{2\hbar}\sum_j (2V_j\rho(t)V_j^\dagger - \{\rho(t), V_j^\dagger V_j\}_+)$$

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{m}{2}(\omega_1^2 x^2 + \omega_2^2 y^2) + qxy$$

In impulse coordinate form for two coupled modes in a thermal environment.

$$\dot{\rho} = \sum_{k=1}^{2} \frac{\lambda}{2}\Big\{(N+1)\mathcal{L}[a_k] + N\mathcal{L}[a_k^\dagger] - M^*\mathcal{D}[a_k] - M\mathcal{D}[a_k^\dagger]\Big\}\rho$$

By using creation and annihilation operators for two non coupled modes in a thermal squeezed environment.

System evolution

$$\frac{d\sigma(t)}{dt} = Z\sigma(t) + \sigma(t)Z^{\mathrm{T}} + 2D$$

$$Z = \begin{pmatrix} -\lambda & 1/m & 0 & 0 \\ -m\omega_1^2 & -\lambda & -q & 0 \\ 0 & 0 & -\lambda & 1/m \\ -q & 0 & -m\omega_2^2 & -\lambda \end{pmatrix} \qquad D = \begin{pmatrix} \frac{\lambda}{2m\omega_1} & 0 & 0 & 0 \\ 0 & \frac{\lambda m\omega_1}{2} & 0 & 0 \\ 0 & 0 & \frac{\lambda}{2m\omega_2} & 0 \\ 0 & 0 & 0 & \frac{\lambda m\omega_2}{2} \end{pmatrix}$$

$$Z\sigma(\infty) + \sigma(\infty)Z^{\mathrm{T}} = -2D$$

$$\sigma(t) = S(t)[\sigma(0) - \sigma(\infty)]S^{\mathrm{T}}(t) + \sigma(\infty),$$

$$S = \exp(Zt)$$

Fidelity of teleportation

$$\mathcal{F}(\rho_1, \rho_2) = \left[\mathrm{Tr}\left(\sqrt{\sqrt{\rho_2}\rho_1\sqrt{\rho_2}}\right)\right]^2$$ - the definition of the fidelity

Resource state: $$\sigma(t) = \begin{pmatrix} A & C \\ C^{\mathrm{T}} & B \end{pmatrix}$$

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \quad B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \quad C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

The received state:

$$\sigma_{out} = \begin{pmatrix} \frac{1}{2} + A_{11} + B_{11} - 2C_{11} & C_{21} + B_{12} - A_{12} - C_{12} \\ C_{21} + B_{12} - A_{12} - C_{12} & \frac{1}{2} + A_{22} + B_{22} + 2C_{22} \end{pmatrix}$$

The final formula for fidelity of teleportation calculation:

$$F = \frac{1}{\sqrt{\det \sigma_{out} + \frac{1}{2}\,\mathrm{tr}\,\sigma_{out} + \frac{1}{4}}}$$

Entanglement

Is a quantum correlation.

Bonds quantum particles without physical interaction.

For successful teleportation the state needs to be entangled. $r > r_e$

r is squeezing parameter

$$\cosh^2 r_e = \frac{(n_1+1)(n_2+1)}{n_1+n_2+1}$$

n_1 and n_2 The number of thermal photons

Logarithmic negativity, the measurement of the entanglement:

$$E(t) = \max\{0, -\frac{1}{2}\log_2[4f(t)]\}$$

Where:

$$f(t) = \frac{1}{2}(\det A + \det B) - \det C - \left(\left[\frac{1}{2}(\det A + \det B) - \det C\right]^2 - \det\sigma(t)\right)^{\frac{1}{2}}$$

Resonant Case

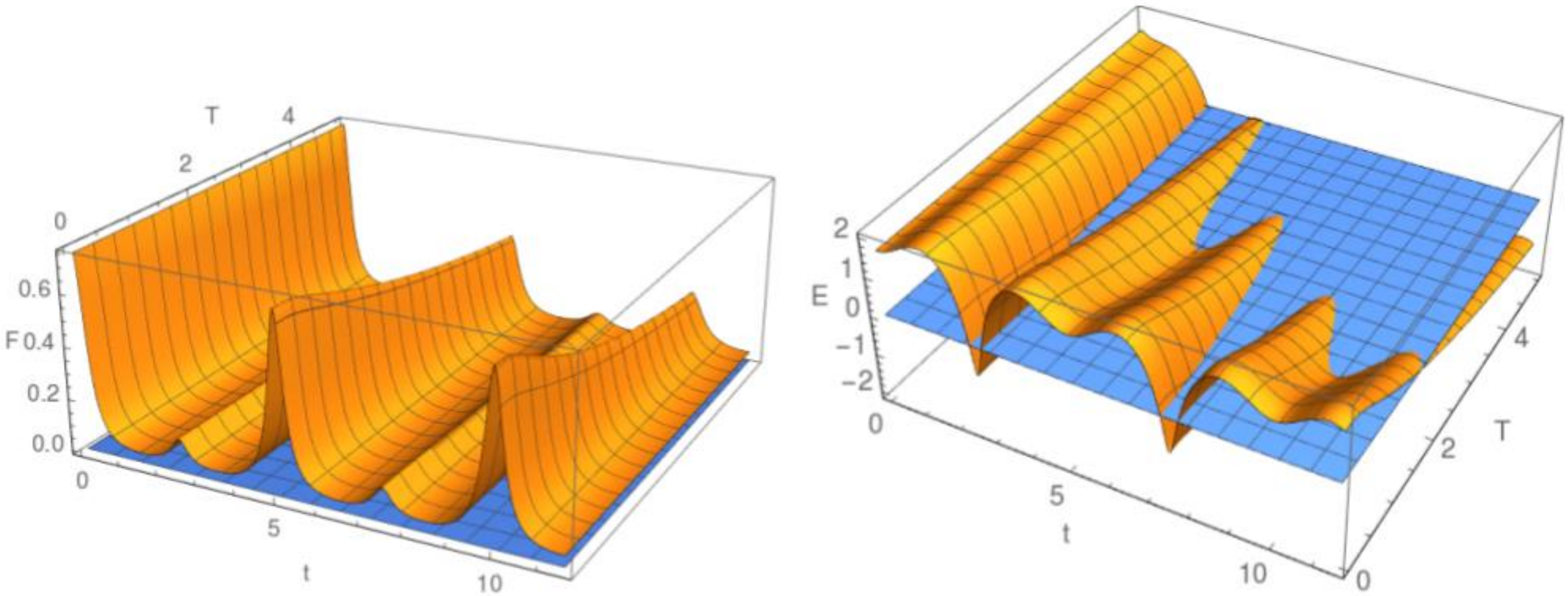


Fig. 1 – : Fidelity of teleportation F (left) and logarithmic negativity E (right) versus time t and temperature T, for $q = 0.6$, $r = 1$, $n_1 = 1$ and $n_2 = 0.5$.

Resonant Case

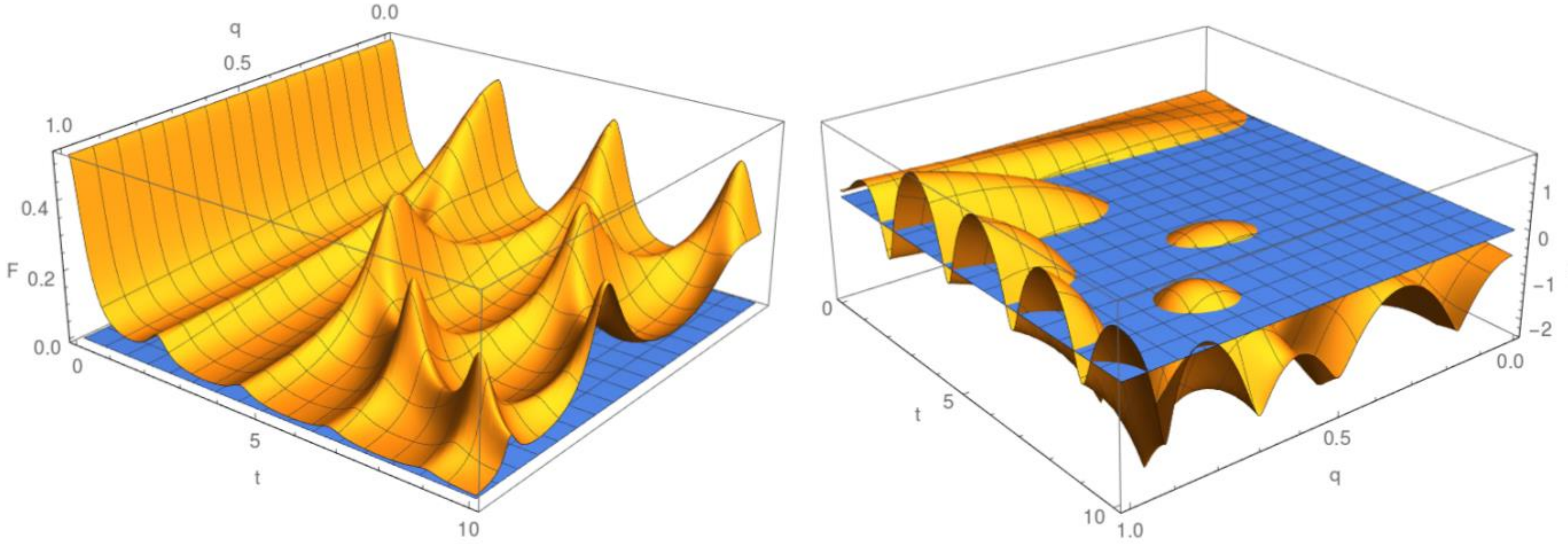


Fig. 3 – : Fidelity of teleportation F (left) and logarithmic negativity E (right) versus time t and coupling q, for $r = 0.6$, $n_1 = n_2 = 1$ and $T = 2$.

Non-Resonant Case

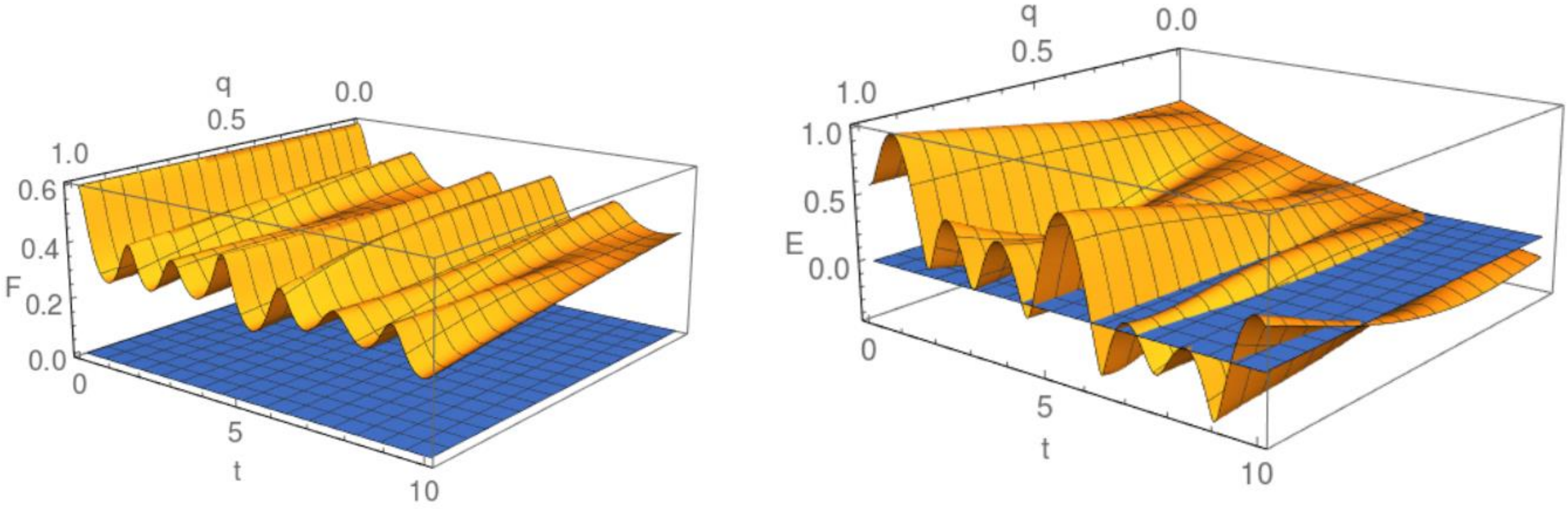


Fig. 5 – : Fidelity of teleportation F (left) and logarithmic negativity E (right) versus time t and coupling q, for $\omega_1 = 1$, $\omega_2 = 2$, $r = 0.2$, $n_1 = n_2 = 0$ and $T = 2$.

Non-Resonant Case

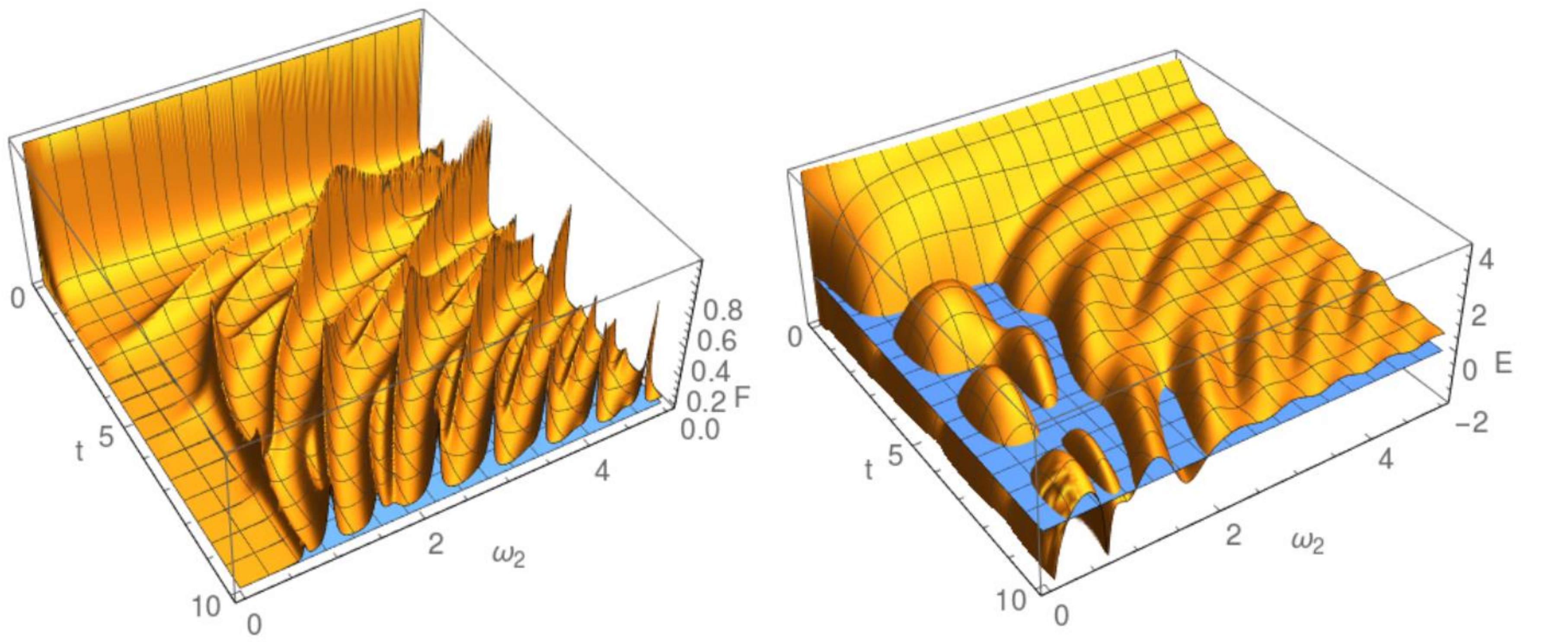
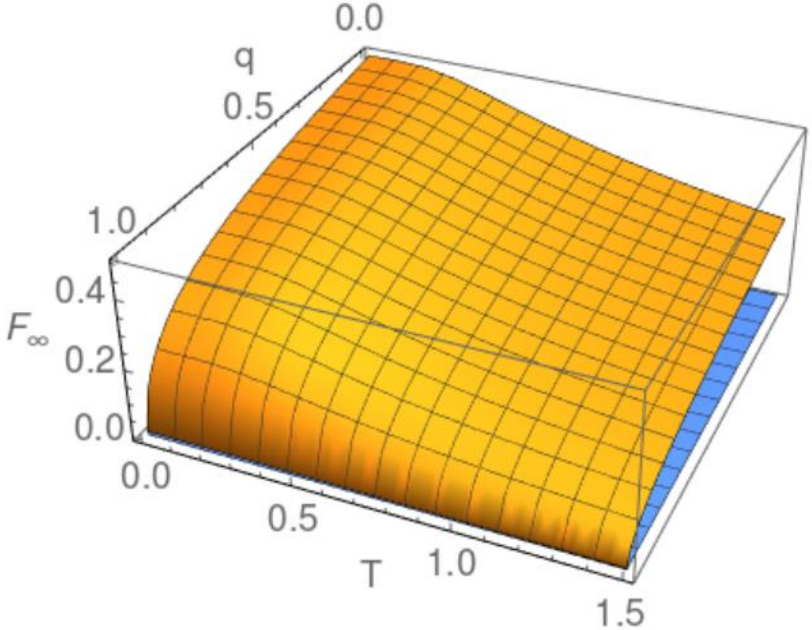
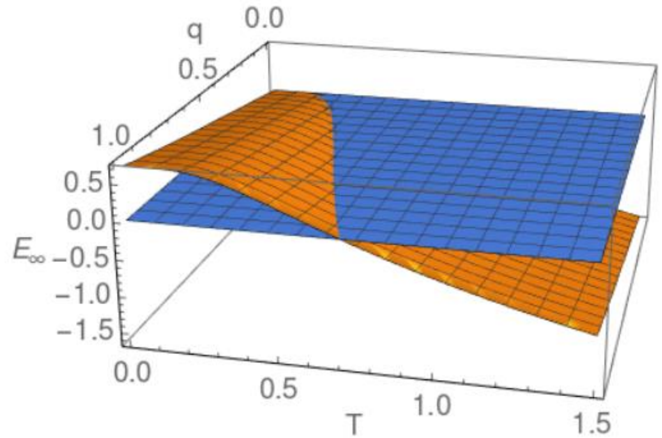


Fig. 7 – : Fidelity of teleportation F (left) and logarithmic negativity E (right) versus time t and the frequency ω_2, for $\omega_1 = 1$, $q = 0.6$, $r = 1.5$, $n_1 = n_2 = 0$ and $T = 2$.

Asymptotic Case

$$\mathcal{F}_\infty = 2\sqrt{\frac{L^2 - q^2}{(4L + 5q + q^2)(4L - 3q)}} \qquad E_\infty = -\frac{1}{2}\log_2\left(1 + \frac{3q^2 L}{4(L^2 - q^2)} - \frac{\sqrt{q^6 + 16q^2L^3 + 8q^4L(L-2)}}{4(L^2-q^2)}\right)$$





Asymptotic fidelity of teleportation *F* (left) and asymptotic logarithmic negativity *E* (right) Versus coupling *q* and temperature *T.*

Conclusions

- The fidelity of teleportation and the entanglement of the resource states strongly depend on the temperature, the initial entangled Gaussian state, the strength of coupling between the two modes, and the frequencies of the modes.
- The fidelity of teleportation can reach values larger than the classical limit 1/2 and larger even than the secure teleportation limit 2/3.
- In the limit of large times, the asymptotic resource state is separable for high temperatures.
- For low temperatures, the asymptotic state remains entangled and, therefore, the quantum teleportation can be realized.

Thank You for Your Attention!!!