The quantum dynamics of a two-qubit pair longitudinally coupled with a single-mode boson field

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OUTLINE:

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- Introduction

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- The model: Approximations

3
- Results and Conclusions

Motivation

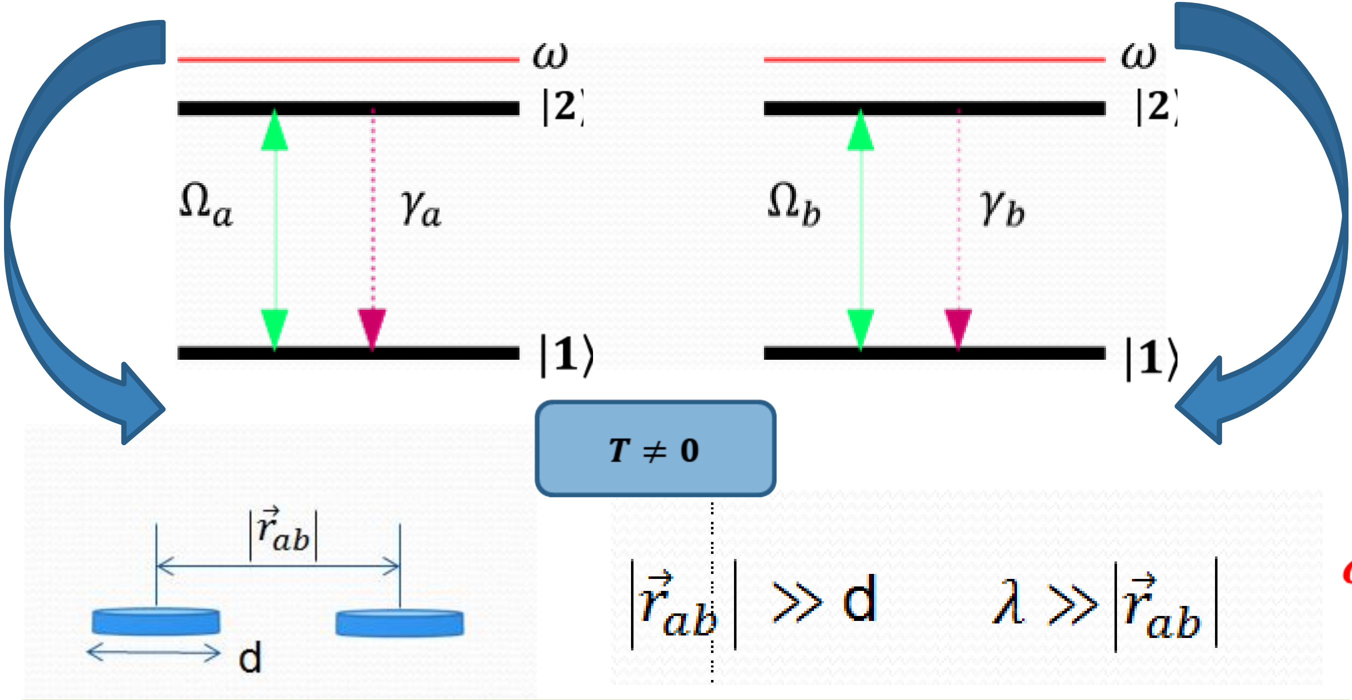
M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, 2000).

Quantum technologies; for instance, quantum sensors, commutators, diodes ,etc,.

H. J. Kimble, The quantum internet, Nature 453, 1023 (2008).

A. Streltsov, G. Adesso and M. B. Plenio, Colloquium: Quantum coherence as a resource, Rev. Mod. Phys. 89, 041003 (2017).

The Model



$\Omega_a = \Omega_b = \Omega$

$\gamma_a = \gamma_b = \gamma$

$\omega_{01} = \omega_{02} = \omega_0 \equiv \omega_L$

E. Cecoi, V. Ciornea, A. Isar, M. A. Macovei, arxiv:1907.11396

SCOPE

- Investigation of the relationship among the cooling effects and entanglement in a pumped pair of two-level qubits longitudinally coupled with a boson mode.

The Hamiltonian

$$\bar{H} = \hbar\Omega_{dd}(S_a^+ S_b^- + S_b^+ S_a^-) + \hbar\Omega(S_a^+ + S_b^+ + H.c.)$$

$$+ \hbar g(S_z^{(a)} + S_z^{(b)})(b + b^\dagger) + \hbar\omega b b^\dagger$$

inter.dipole-dipole from Q

inter.Q+ phonons

inter.laser + Q

Energy phonons

Z. Ficek, S. Swain, *Quantum Interference and Coherence* (Springer, Berlin 2005)

The master equation

$$\dot{\rho}+\frac{i}{\hbar}[\bar{H},\rho]=-\frac{\gamma}{2}\{[S_a^+,S_a^-\rho]+[S_b^+,S_b^-\rho]\}-\frac{\gamma}{2}\chi_{ab}\{[S_a^+,S_b^-\rho]+[S_b^+,S_a^-\rho]\}$$

$$-\frac{k}{2}(1+\bar{n})[b^\dagger,b\rho]-\frac{k}{2}\bar{n}\,[b,b^\dagger\rho]+H.c.,$$

$$\bar{H}=H+H_i,$$

$$H=\hbar\omega bb^\dagger+\hbar g\sum_{j\in\{q1,q2\}}S_z^j(b+b^\dagger),$$

$$H_i=\hbar\Omega_{dd}\sum_{j\neq l\in\{q1,q2\}}S_j^+S_l^-+\hbar\Omega\sum_{j\in\{q1,q2\}}(S_j^++S_l^-).$$

Diagonalizeation H_i → $|2_{q1}2_{q2}\rangle$, $|2_{q1}1_{q2}\rangle$, $|1_{q1}2_{q2}\rangle$, $|1_{q1}1_{q2}\rangle$

$\Delta=0$

Z. Ficek, S. Swain, *Quantum Interference and Coherence* (Springer, Berlin 2005)

Eigenfunctions vs eigenvalues

$$|\Psi_4\rangle = -\bar{a}\{|2_{q1}2_{q2}\rangle + |1_{q1}1_{q2}\rangle\} + \bar{b}\{|2_{q1}1_{q2}\rangle + |1_{q1}2_{q2}\rangle\},$$

$$|\Psi_3\rangle = -\bar{c}\{|2_{q1}2_{q2}\rangle + |1_{q1}1_{q2}\rangle\} + \bar{d}\{|2_{q1}1_{q2}\rangle + |1_{q1}2_{q2}\rangle\},$$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\{|2_{q1}1_{q2}\rangle - |1_{q1}2_{q2}\rangle\},$$

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\{|2_{q1}2_{q2}\rangle - |1_{q1}1_{q2}\rangle\}.$$

here

$$\bar{a} = \frac{(\Omega_{dd}-\lambda_4)/\sqrt{2}}{\sqrt{(\Omega_{dd}-\lambda_4)^2+4\Omega^2}}, \qquad \bar{b} = \sqrt{\frac{2\Omega^2}{\sqrt{(\Omega_{dd}-\lambda_4)^2+4\Omega^2}}},$$

$$\bar{c} = \frac{(\Omega_{dd}-\lambda_3)/\sqrt{2}}{\sqrt{(\Omega_{dd}-\lambda_3)^2+4\Omega^2}}, \qquad \bar{d} = \sqrt{\frac{2\Omega^2}{\sqrt{(\Omega_{dd}-\lambda_3)^2+4\Omega^2}}},$$

with

$$\lambda_1 = 0$$

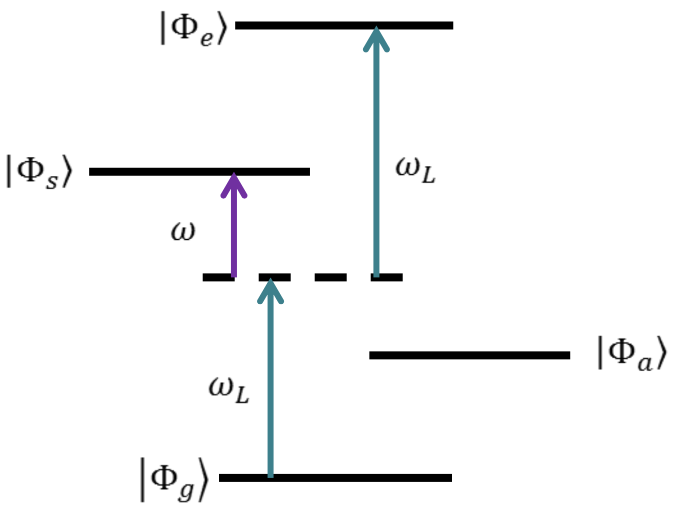
$$\lambda_2 = -\Omega_{dd}$$

$$\lambda_3 = (\Omega_{dd} + \sqrt{\Omega_{dd}^2 + 16\Omega^2})/2$$

$$\lambda_4 = (\Omega_{dd} - \sqrt{\Omega_{dd}^2 + 16\Omega^2})/2$$

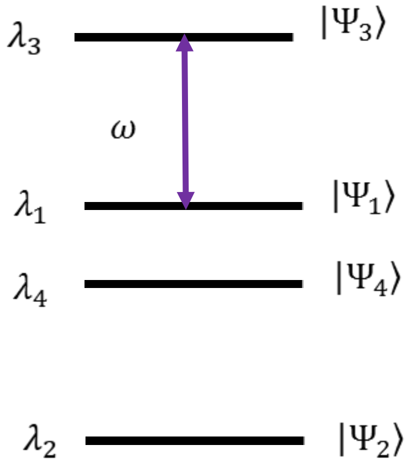
The two-qubit cooperative states

DICKE STATES



$\omega_0 \equiv \omega_L$

DRESSED-STATES



$|\Phi_g\rangle \leftrightarrow |\Phi_s\rangle \leftrightarrow |\Phi_e\rangle$ or $|\Phi_g\rangle \leftrightarrow |\Phi_e\rangle$

$\lambda_1, \lambda_2, \lambda_3, \lambda_4$ *- eigenvalues*

$|\Psi_1\rangle, |\Psi_2\rangle, |\Psi_3\rangle, |\Psi_4\rangle$ *- eigenfunctions*

The model in dressed-states

$$\bar{H}_0 = \hbar\lambda_4 R_{44} - \hbar\delta b^\dagger b - \hbar\bar{g}(R_{31}b + b^\dagger R_{13})$$

$$[b, b^\dagger] = 1$$

$$[b, b\,] = [b^\dagger, b^\dagger] = 0$$

$$\delta = \lambda_3 - \omega$$

$$R_{\alpha\beta} = |\alpha\rangle\ \langle\beta|,$$

$$\{\alpha, \beta\} = \{|\Psi_1\rangle, |\Psi_2\rangle, |\Psi_3\rangle, |\Psi_4\rangle\}.$$

Aproximations

1. $\omega \gg \sqrt{2}g$
2. $\Omega_{dd} \gg \gamma$

THE FINAL MASTER EQUATION IN DRESSED-STATES

$$\dot{\rho} + \frac{i}{\hbar}[\bar{H}_0, \rho] = -\frac{\gamma}{2}(1+\chi_r)\Big(2\Big[\bar{c}\bar{d}R_{44} + \bar{a}\bar{b}R_{33} + \frac{\bar{c}}{2\sqrt{2}}(R_{41} - R_{14}), \Big\{4\Big(\bar{c}\bar{d}R_{44} + \bar{a}\bar{b}R_{33} + \frac{\bar{c}}{2\sqrt{2}}(R_{14} - R_{41})\Big)\Big\}\rho\Big]$$

$$+2(\bar{a}\bar{d}+\bar{b}\bar{c})^2\{[R_{34},R_{43}\rho] + [R_{43},R_{34}\rho]\} + \bar{a}^2\{[R_{13},R_{31}\rho] + [R_{31},R_{13}\rho]\} - \sqrt{2}\bar{a}(\bar{a}\bar{d}+\bar{b}\bar{c})\{[R_{43},R_{31}\rho] + [R_{13},R_{34}\rho] - [R_{34}R_{13}\rho]$$

$$-[R_{31},R_{43}\rho]\}\Big) - \frac{\gamma}{2}(1-\chi_r)(\bar{b}^2\{[R_{32},R_{23}\rho] + [R_{23},R_{32}\rho]\} + \bar{d}^2\{[R_{24},R_{42}\rho] + [R_{42},R_{24}\rho]\} + \frac{1}{2}\{[R_{12},R_{21}\rho] + [R_{21},R_{12}\rho]\}$$

$$-\frac{\bar{d}}{\sqrt{2}}\{[R_{42},R_{21}\rho] + [R_{12},R_{24}\rho] - [R_{24},R_{12}\rho] - [R_{21},R_{42}\rho]\}\Big) - \frac{k}{2}(1+\bar{n})[b^\dagger, b\rho] - \frac{k}{2}\bar{n}[b, b^\dagger\rho] + H.c.$$

The equations of motion (I)

$$\frac{d}{dt}\rho_{22} = i\delta\{b^\dagger b\rho_{22} - \rho_{22}b^\dagger b\} + \frac{\gamma}{2}(1-\chi_r)\rho_{11} - \left\{\gamma d^2(1-\chi_r) + \gamma b^2(1-\chi_r) + \frac{\gamma}{2}(1-\chi_r)\right\}\rho_{22}$$

$$+\ \gamma b^2(1-\chi_r)\rho_{33} + \gamma d^2(1-\chi_r)\rho_{44} - \frac{\gamma}{\sqrt{2}}d(1-\chi_r)\{\rho_{14}+\rho_{41}\}$$

$$-\frac{k}{2}\,(1+\bar{n})\{b^\dagger b\rho_{22} - 2b\rho_{22}b^\dagger + \rho_{22}b^\dagger b\} - \ \frac{k}{2}\bar{n}\,\{bb^\dagger\rho_{22} - 2b^\dagger\rho_{22}b + \rho_{22}bb^\dagger\}.$$

$$\boldsymbol{\rho_{\alpha\beta} = \langle \Psi_\alpha | \rho | \Psi_\beta \rangle}$$

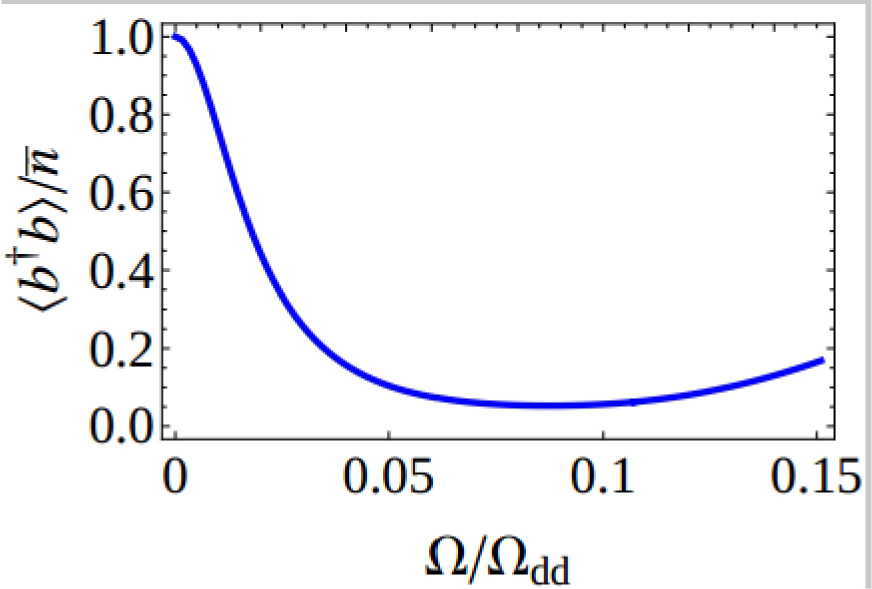
The projection in the Fock state basis

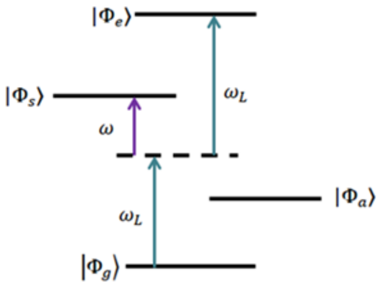
$$P_n^{(i)} = \langle n|\rho^{(i)}|n\rangle, \{|n\rangle, n \in \mathcal{N}\}$$

$$\frac{d}{dt}P_n^{(2)} = -k(1+\bar{n})(nP_n^{(2)} - (n+1)P_{n+1}^{(2)}) - k\bar{n}((n+1)\,P_n^{(2)} - nP_{n-1}^{(2)}) + \gamma_0^{(2)}P_n^{(0)} + \gamma_1^{(2)}P_n^{(1)} - \gamma_2^{(2)}P_n^{(2)}$$
$$+\gamma_3^{(2)}P_n^{(3)} - \gamma_{11}^{(2)}P_n^{(11)},$$

$$b^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$$
$$b|n\rangle = \sqrt{n}\,|n-1\rangle$$

Results (I)



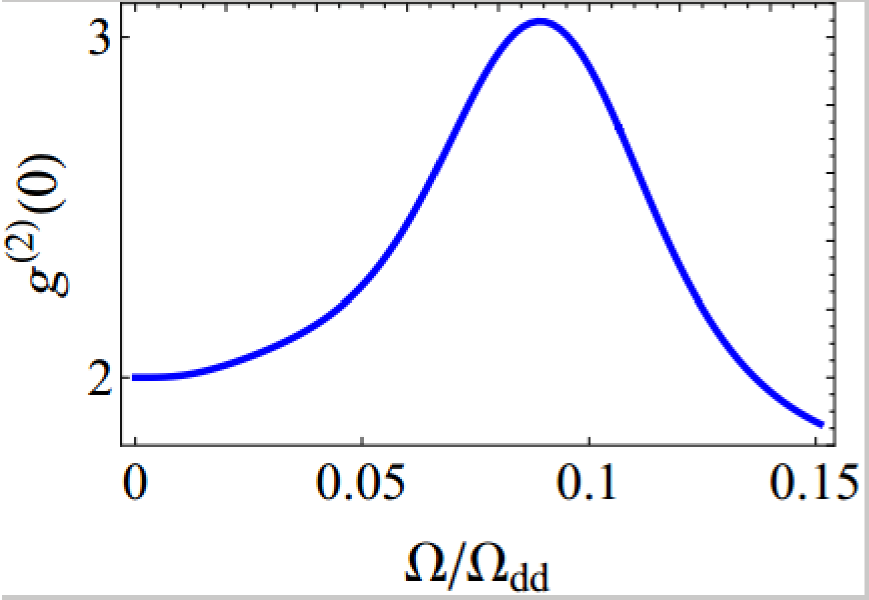


The dependence of the mean phonon number as a function of scaled Rabi frequency

$\bar{n}$ -mean phonons number due to thermal reservoir

$$g/\gamma = 2,\ \Omega_{dd}/\gamma = 28,\ \omega/\gamma = 30,\ \chi_r = 0.98,\ \bar{n} = 20\quad \kappa/\gamma = 10^{-3}.$$

Results (II)



Super-Poissonian phonon statistics

The dependence of $g^{(2)}(0)$ vs Ω/Ω_{dd}

$g/\gamma = 2,\ \Omega_{dd}/\gamma = 28,\ \omega/\gamma = 30,\ \chi_r = 0.98,\ \bar{n} = 20\quad \kappa/\gamma = 10^{-3}.$

ENTANGLEMENT: Concurrence

$$C = \max\left\{0,\ s_1 - \sum_{\xi=2}^{4} s_\xi\right\},$$

$$Q = \tilde{\rho}_{q1q2}\left(\sigma_{q1y} \otimes \sigma_{q2y}\right)\tilde{\rho}^{*}{}_{q1q2}\left(\sigma_{q1y} \otimes \sigma_{q2y}\right)$$

$|2_{q1}2_{q2}\rangle$, $|2_{q1}1_{q2}\rangle$, $|1_{q1}2_{q2}\rangle$, $|1_{q1}1_{q2}\rangle$

$$\tilde{\rho}_{q1q2} = \begin{bmatrix} \tilde{\rho}_{11} & \tilde{\rho}_{12} & \tilde{\rho}_{13} & \tilde{\rho}_{14} \\ \tilde{\rho}_{21} & \tilde{\rho}_{22} & \tilde{\rho}_{23} & \tilde{\rho}_{24} \\ \tilde{\rho}_{31} & \tilde{\rho}_{32} & \tilde{\rho}_{33} & \tilde{\rho}_{34} \\ \tilde{\rho}_{41} & \tilde{\rho}_{42} & \tilde{\rho}_{43} & \tilde{\rho}_{44} \end{bmatrix}$$

$$\sigma_{q1y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

S. Hill, and W. K. Wootters, Entanglement of a pair of quantum bits, Phys. Rev. Lett. **78**, 5022 (1997);
W. K. Wootters, Entanglement of formation of an arbitrary state of two qubits, Phys. Rev. Lett. **80**, 2245 (1998).

The variables of interest

$$\tilde{\boldsymbol{\rho}}_{11} = S_{ee}^{(a)} \cdot S_{ee}^{(b)}$$

$$S_{ee}^{(a)} = \frac{1}{2} + \frac{\xi}{\sqrt{2}}\{R_{42} + R_{24} + R_{31} + R_{13}\} + \frac{\eta}{\sqrt{2}}\{R_{32} + R_{23} - R_{41} - R_{41}\}$$

$$S_{ee}^{(b)} = \frac{1}{2} + \frac{\xi}{\sqrt{2}}\{R_{31} + R_{13} - R_{42} - R_{24}\} - \frac{\eta}{\sqrt{2}}\{R_{41} + R_{14} + R_{32} + R_{23}\}$$

$$\tilde{\boldsymbol{\rho}}_{11} = \frac{1}{4} - \frac{\eta}{\sqrt{2}}(\rho_{41} + \rho_{14}) - \frac{\Omega_{dd}}{4\sqrt{\Omega_{dd}{}^{2} + 16\Omega^{2}}}(\rho_{33} - \rho_{44}) + \frac{1}{4}(\rho_{11} - \rho_{22}).$$

$\xi, \eta = f(\Omega_{dd}, \Omega)$

The variables of interest

$$\tilde{\rho}_{11} = \frac{1}{4}(1+\rho_{11}-\rho_{22}) - \frac{\Omega_{dd}}{4\sqrt{\Omega_{dd}^2+(4\Omega)^2}}(\rho_{33}-\rho_{44}) - \frac{\bar{a}}{\sqrt{2}}(\rho_{41}+\rho_{14}),$$

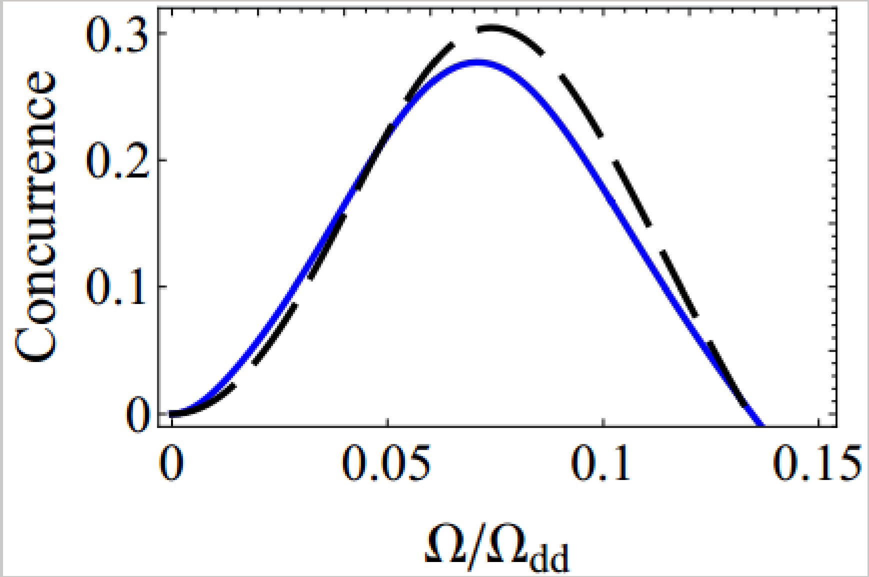
$$\tilde{\rho}_{12} = \frac{\Omega}{\sqrt{\Omega_{dd}^2+(4\Omega)^2}}(\rho_{33}-\rho_{44}) - \frac{\bar{c}}{\sqrt{2}}\rho_{41},$$

$$\tilde{\rho}_{22} = \frac{1}{4}(1+\rho_{22}-\rho_{11}) + \frac{\Omega_{dd}}{4\sqrt{\Omega_{dd}^2+(4\Omega)^2}}(\rho_{33}-\rho_{44}),$$

$$\tilde{\rho}_{23} = \frac{1}{4}(1-\frac{\Omega_{dd}}{\sqrt{\Omega_{dd}^2+(4\Omega)^2}})\rho_{44} - \frac{1}{2}\rho_{22} + \frac{1}{4}(1+\frac{\Omega_{dd}}{\sqrt{\Omega_{dd}^2+(4\Omega)^2}})\rho_{33},$$

$$\tilde{\rho}_{13} = \tilde{\rho}_{12}, \qquad \tilde{\rho}_{21} = (\tilde{\rho}_{12})^{\dagger},$$

Results III

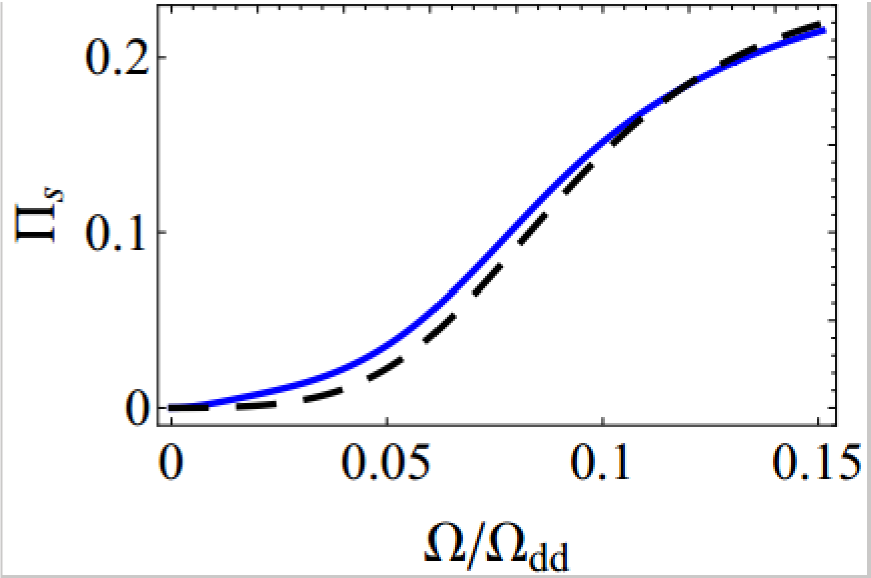


$g \neq 0$ (solid line)

$g = 0,$ (dashed line)

$g/\gamma = 2,\ \Omega_{dd}/\gamma = 28,\ \omega/\gamma = 30,\ \chi_r = 0.98,\ \bar{n} = 20\ \ \kappa/\gamma = 10^{-3}.$

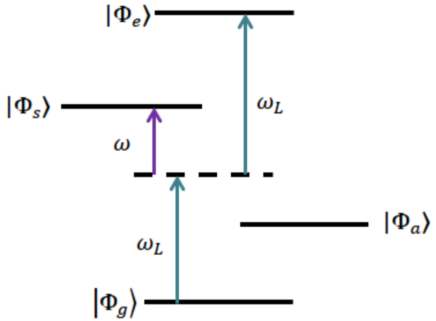
Results IV



$$\Pi_s = \langle |\Phi_s\rangle\langle\Phi_s| \rangle,$$

$g \neq 0$ (solid line)

$g = 0,$ (dashed line)



$g/\gamma = 2,\ \Omega_{dd}/\gamma = 28,\ \omega/\gamma = 30,\ \chi_r = 0.98,\ \bar{n} = 20\ \ \kappa/\gamma = 10^{-3}.$

Conclusions

I. We have demonstrated the relationship among the entanglement creation in a laser-pumped dipole-dipole interacting two-level qubits and the cooling effects of a boson mode which is longitudinally coupled with the both quantum emitters.

II. The cooling occurs on the expenses of the entanglement among the two qubits.

THANKS!